Equations and images expressed by a Hyper complex

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Abstract

We have heard the number can be designed freely under the field that satisfied 4 arithmetic operations. We introduce number-systems having plural imaginary axes, in which are derived from "kaleidoscopic" primes. The operational logic is kept like as the complex field. Some images connected to equations are found.

Keywords: Hyper complex, imaginary axis, absolute, ellipsoid, Apollonius circle

1. ω-number

We define a set $\{X,Y\}$ whose expressions are,

$\{\omega^3=1,\ \omega^2+\omega+1=0\}$, $\omega=(-1+3^{0.5}i)/2$; $X=a+b\omega$, $Y=c+d\omega$. (1)

Where $\{a,b,c,d\}$ is the real variables, and "i" is imaginary unit of the complex [1].

We confirm following relations,

$X\pm Y=(a+c)\pm(b+d)\omega$; $XY=(ac-bd)+(ad+bc-bd)\omega$, $X^2=a^2+2ab+b^2\omega^2$; (2)

$Conj(a+b\omega)=a+b\omega^2=(a-b)-b\omega$; $X/Y=\{(ac-bd-ad)+(bc-ad)\omega\}/(c^2+d^2-cd)$; (3)

$X\ Conj(X)=(a+b\omega)(a+b\omega^2)=a^2+b^2-ab$ =real number :=$|X|$; (4)

$|X|=a^2+b^2-2ab+ab=(a-b)^2+ab$, $if(quadrant\ 1/3)\ 0<|X|$, and $|X|=a^2+b^2-ab$, $if(quadrant\ 2/4)\ 0<|X|$,

Therefore, $0<|X|$, for any (a,b). (4A)

Considering above relations, $\{X,Y\}$ is a field; we call it ω-number. It has 2 imaginary axes.

2. Equations of ω-number

An equation, $X^2+Y^2=\mathrm{R}^2$, R is on real axis (constant), (5)

$X^2+Y^2=a^2+c^2+2(ab+cd)+(b^2+d^2)\omega^2=\mathrm{R}^2$,

$b^2+d^2=0$, and $a^2+c^2+2(ab+cd)=\mathrm{R}^2$,

$b=d=0$, $a^2+c^2=\mathrm{R}^2$; thus, Eq.(5) gives a circle same as the real number.

An equation, $X^2+Y^2=\omega\mathrm{R}^2$, on ω-axis, (6)

$X^2+Y^2=a^2+c^2+2(ab+cd)-(b^2+d^2)(1+\omega)=\omega\mathrm{R}^2$,

$a^2+c^2+2(ab+cd)-b^2-d^2=0$, and $-(b^2+d^2)=\mathrm{R}^2$, there is no image.

An equation, $X^2+Y^2=\omega^2\mathrm{R}^2$, on ω^2-axis, (7)

$X^2+Y^2=a^2+c^2+2(ab+cd)+(b^2+d^2)\omega^2=\omega^2\mathrm{R}^2$,

$a^2+c^2+2(ab+cd)=0$, and $b^2+d^2=\mathrm{R}^2$. (7A)

An image of Eq(7A) is displayed by CG whose resolution 721×721 pixels (Figure 1).

Apollonius circle [2] is given by an equation, $|X|=\mathrm{R}^2$, (8)

$a^2+b^2-ab=\mathrm{R}^2$. It is not a circle but an ellipse that has an axis of 135 deg.

3. Extension of ω-number

The ω-number can be extended as followings;

$\{\omega^5=1,\ \omega^4+\omega^3+\omega^2+\omega+1=0\}$; $Conj(\omega^4)=\omega$, $Conj(\omega^3)=\omega^2$, $X=a+b\omega+c\omega^2+c3\omega^3$, $Y=d+e\omega+f\omega^2+f3\omega^3$, (9)

where $\{a\sim f,c3,f3\}$ is the real variables. We call them ω^5-number, whose imaginary axis is 4. On the definition of Eq.(9), next

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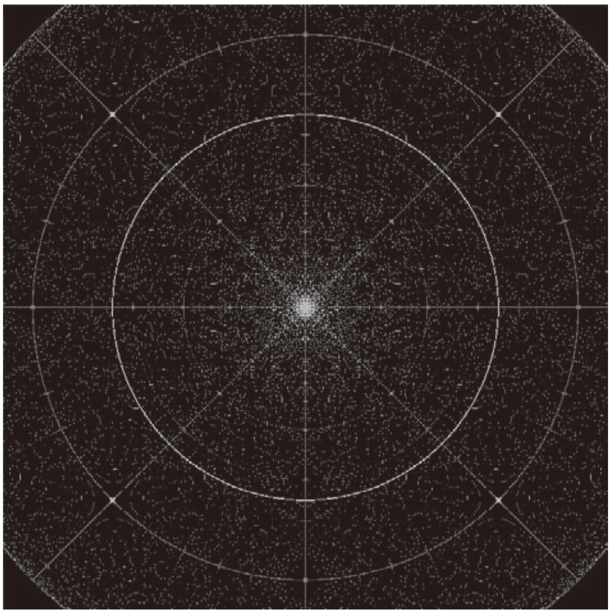

Figure 1.
$\{(a,c),(b,d)\}$ image of "$a^2+c^2+2(ab+cd)=0$, *and* $b^2+d^2=1$"
The pain is $[-\pi/2,\pi/2]$ for horizontal and virtual axes. The axis is constructed with 721 dots. Set of (a,c) is drawn by red dots, and (b,d) is blue (it is a circle.). Red pattern is constructed by radial lines and concentric circles. Red tiny dots is risen by finite calculations. The precisions are $|b^2+d^2-1|< 0.7\pi/721$ and $|a^2+c^2+2(ab+cd)|< 0.1\pi/721$.
The red-dot distribution indicates a new research field; *i.e.*, "investigation of spaces nearby the pole".

expressions are derived.

$X\pm Y=(a\pm d)+(b\pm e)\omega+(c\pm f)\omega^2+(c3\pm f3)\omega^3$,

$XY=(ad+cf3+c3f-w)+(ae+bd+c3f3-w)\omega+(af+be+cd-w)\omega^2+(af3+bf+ce+c3d-w)\omega^3$, $w=(bf3+cf+c3e)$, (10)

$Conj(a+b\omega+c\omega^2)=a+b\omega^4+c\omega^3$,

$X/Y=(a+b\omega+c\omega^2)(d+e\omega^4+f\omega^3)/\{(d+e\omega+f\omega^2)(d+e\omega^4+f\omega^3)\}$, (11)

The denominator (D) of Eq.(11) is $D=(d^2+e^2+f^2)+(de+ef)(\omega+\omega^4)+df(\omega^2+\omega^3)$:=real number. Thus, X/Y is a number of ω^5-number. The absolute, $|X|=X\,Conj(X)=$
$(a+b\omega+c\omega^2+d\omega^3)(a+b\omega^4+c\omega^3+d\omega^2)=(a^2+b^2+c^2+d^2)+(-1+5^{0.5})(ab+bc+cd)/4-(1+5^{0.5})(ad+ac+bd)/4$, because of $(\omega+\omega^4)=(-1+5^{0.5})/4\sim0.309$, $(\omega^2+\omega^3)=-(1+5^{0.5})/4\sim-0.809$, where $(1+5^{0.5})/2$ is golden ratio. (12)

Introducing a real number "A", we get; $\mathrm{I}=(a^2+b^2)+Aab=(a+b)^2+(A-2)ab=\mathrm{II}$; if$\{0<A$, and $(A-2)<0$; $0<A<2\}$then $0<\{\mathrm{I},\mathrm{II}\}$.
$\mathrm{I}=(a^2+b^2)-Aab=(a+b)^2+(-A-2)ab=\mathrm{II}$; if$\{A<0$, and $(-A-2)<0$; $-2<A<0\}$then $0<\{\mathrm{I},\mathrm{II}\}$.

Thus; $0 < |X|$ is proved, and the absolute is a positive real-number in the extension. The Apollonius circle is existed corresponding with Eq.(8). It is not a closed line but 4-dimensional ellipsoid. The surface is displayed by CG (Figure 2).

The following is about coloring of the ring. To make of color rings, you must consider threshold (TH) and the number (N) of pixels of display. An example is N=721, $TH=0.05/N$. Especially, TH must be changed for generation of patterns. An equation, $I=(x^2+y^2+z^2)-(xy+yz+zx)(1+5^{0.5})/8$, $-\pi/4<\{x,y,z\}<\pi/4$, is 3D ellipsoid.

Pixel address is $Ad(x)=[x/dx], dx=(\pi/8)/N$. Imaging function is, $|I-r(R)|<TH$, $|I-r(B)|<TH$, $|I-r(G)|<TH$; $\{r(R),r(B),r(G)\}=\{\pi/4+dx, \pi/4, \pi/4-dx\}$. Generation image is in Figure 3.

Newton ring is arisen from small difference of optical path. That of CG is the bit-length of registers, the bit-step function. Therefore, edges of patterns in Figure 3 are sharp and do not change gradually. This is a different point. Using an ideal case, $x^2+y^2+z^2=1$; this is a spherical surface, where two type patterns are found. One has 4 sub-rings, and another has 6 rings (Figure 4). Thus; they are different points apparently. We believe the reason is the location-difference between those points influence calculating precision. The patterns don't indicate the property of equations. They indicate the surface curvature qualitatively. Torus having positive/negative curvatures is in Figure 5.

4. Conclusion

We show 2 number systems that have 2/4 imaginaries. They are the fields, and have the absolute that is a positive real number. It indicates existence of Apollonius circle. It is a new knowledge; they are a kind of the hyper complex. Rings like Newton's are beautiful and they are useful tools for educators.

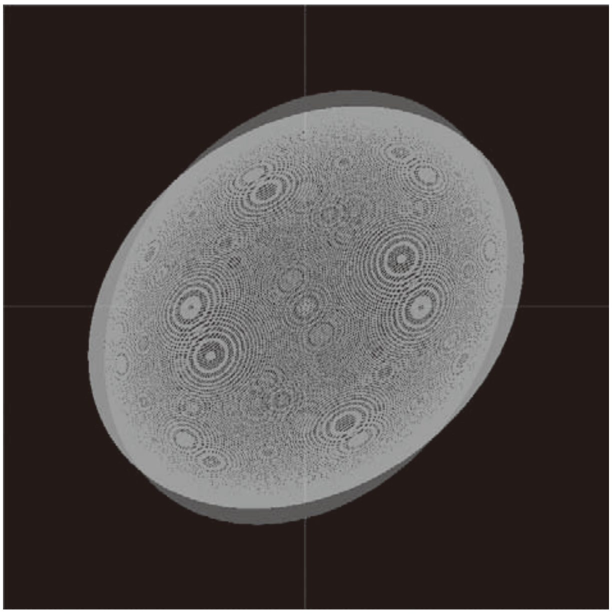

Figure 2. A projection {(a,b),(b,c), d=0} image of |X|=1 in the ω^5-number.
The pain is [-π/2,π/2] for horizontal and vertical axes. Set of (a,b) is drawn by red dots, and (b,c) is blue. The image is the surface of ellipsoid. We represent it by CG (the threshold is 0.7π/721), where patterns like Newton ring [3].

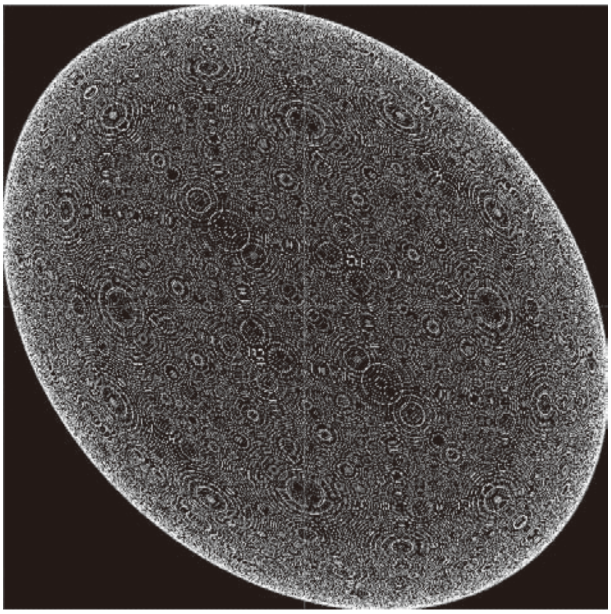

Figure 3. Patterns like Newton ring on the surface of 3D ellipsoid.
The pain is [-π/4,π/4] for horizontal and vertical axes. Set of (x,y) is drawn by red dots, (y,z) is blue, and (z,x) is green. Even if there is no wave; Is that Newton rings? It is wondering.

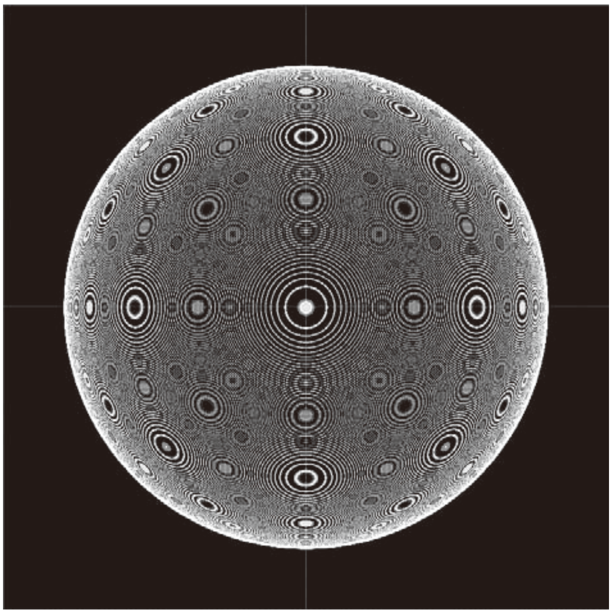

Figure 4. Curvature-rings of spherical, radius=π/8.
The pain is [-π/4,π/4] for horizontal and vertical axes. Set of (x,y) is drawn by red dots, (y,z) is green, and (z,x) is blue.

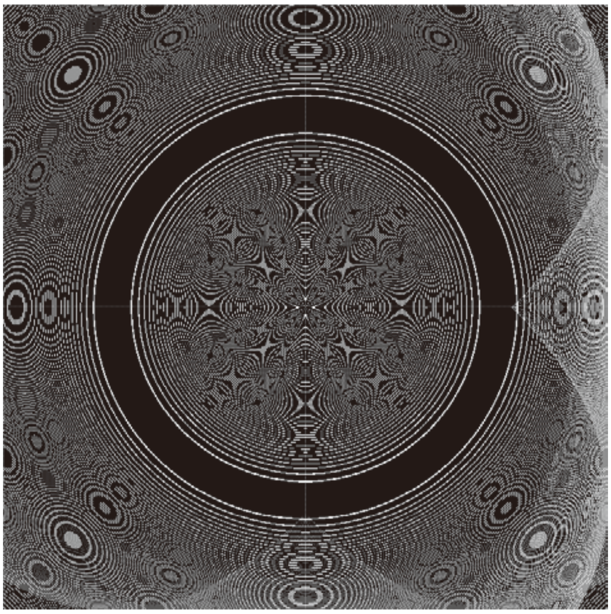

Figure 5. Curvature-rings of torus, $\{(x^2+y^2)^{0.5}-b\}^2+z=r$, $r=\pi/4,\ b=1/2$.
The pain is [-π/4,π/4] for horizontal and vertical axes. Set of (x,y) is drawn by green dots, (y,z) is blue, and (z,x) is red. Bold black ring is curvature=0. Positive curvature zone is outer the ring. Moiré pattern indicates the curvature.

References

[1] Kazuya Kato,"Suron eno Syotai (Japanese)", *Springer mathematical club* vol.23, Maruzen Pub. Co. Ltd., 2012.
This is a text book for elementary course. There is no description of ω-number's property. The "kaleidoscopic" prime numbers are written.

[2] Apollonius circle is expressed by many Web pages; *i.e.*, https://en.wikipedia.org/wiki/Circles_of_Apollonius.

[3] Newton ring, https://en.wikipedia.org/wiki/Newton%27s_rings.